Terme & Typen

- $\tau, \rho ::= \ \Diamond \mid \mathbf{N} \mid \mathrm{bool} \mid \tau \multimap \rho \mid \tau \otimes \rho \mid \tau \times \rho \mid \mathbf{L}(\tau)$
- $t, s ::= \ x \mid c \mid \lambda x^{\tau} t \mid \langle t, s \rangle \mid ts \mid \{t\} \mid \{t, s\}$
- Konstanten:

$\mathbf{t}$	bool
$\mathbf{f}$	bool
0	$\mathbf{N}$
$\mathbf{S}_0$	$\Diamond \multimap \mathbf{N} \multimap \mathbf{N}$
$\mathbf{S}_1$	$\Diamond \multimap \mathbf{N} \multimap \mathbf{N}$
$\mathbf{nil}_{\tau}$	$\mathbf{L}(\tau)$
$\mathbf{cons}_{\tau}$	$\Diamond \multimap \tau \multimap \mathbf{L}(\tau) \multimap \mathbf{L}(\tau)$
$\otimes_{\tau,\rho}$	$\tau \multimap \rho \multimap \tau \otimes \rho$

Lineare Typisierung

$$\frac{}{\Gamma, x^\tau \vdash x^\tau} \qquad (Var)$$

$$\frac{c \text{ vom Typ } \tau}{\Gamma \vdash c^\tau} \qquad (const)$$

$$\frac{\Gamma \cup \{x^\tau\} \vdash t^\rho}{\Gamma \vdash (\lambda x^\tau t)^{\tau \multimap \rho}} \qquad (\multimap^+)$$

$$\frac{\Gamma \vdash t^\tau \qquad \Gamma \vdash s^\rho}{\Gamma \vdash \langle t, s\rangle^{\tau \times \rho}} \qquad (\times^+)$$

$$\frac{\Gamma_1 \vdash t^{\tau \multimap \rho} \qquad \Gamma_2 \vdash s^\tau}{\Gamma_1, \Gamma_2 \vdash (ts)^\rho} \qquad (\multimap^-)$$

$$\frac{\Gamma \vdash t^{\tau \times \rho}}{\Gamma \vdash (t\mathbf{t})^\tau} \qquad (\times_1^-)$$

$$\frac{\Gamma \vdash t^{\tau \times \rho}}{\Gamma \vdash (t\mathbf{f})^\rho} \qquad (\times_0^-)$$

Lineare Typisierung (cont'd)

$$\frac{\Gamma_1 \vdash t^{\mathrm{bool}} \qquad \Gamma_2 \vdash s^\tau \qquad \Gamma_2 \vdash r^\tau}{\Gamma_1, \Gamma_2 \vdash (t\,\langle s, r\rangle)^\tau} \qquad (\mathrm{bool}^-)$$

$$\frac{\Gamma_1 \vdash t^{\tau \otimes \rho} \qquad \Gamma_2, x^\tau, y^\rho \vdash s^\sigma}{\Gamma_1, \Gamma_2 \vdash (t(\lambda x^\tau, y^\rho\, s))^\sigma} \qquad (\otimes^-)$$

$$\frac{\Gamma \vdash t^{\mathbf{N}} \qquad \emptyset \vdash {s_0}^{\diamond \multimap \tau \multimap \tau} \qquad \emptyset \vdash {s_1}^{\diamond \multimap \tau \multimap \tau}}{\Gamma \vdash (t\,\{s_0, s_1\})^{\tau \multimap \tau}} \qquad (\mathbf{N}^-)$$

$$\frac{\Gamma \vdash t^{\mathbf{L}(\tau)} \qquad \emptyset \vdash s^{\diamond \multimap \tau \multimap \rho \multimap \rho}}{\Gamma \vdash (t\,\{s\})^{\rho \multimap \rho}} \qquad (\mathbf{L}(\tau)^-)$$

Sei $\Gamma \vdash t^\tau$. Dann ist t von einer der Formen $x\vec{t}$, $c\vec{t}$, $(\lambda x^\rho s)\vec{t}$ oder $\langle s, r\rangle\,\vec{t}$.

Reduktionsregeln

$$
\begin{array}{ll}
(\lambda x^\tau\, t)s \mapsto t[s/x] & \\
\langle t, s\rangle\, \mathbf{t} \mapsto t & \\
\langle t, s\rangle\, \mathbf{f} \mapsto s & \\
\mathbf{t}\, \langle t, s\rangle \mapsto t & \\
\mathbf{f}\, \langle t, s\rangle \mapsto s & \\
\otimes_{\tau,\rho} ts(\lambda x^\tau, y^\rho\, r) \mapsto r[t, s/x, y] & \\
0\, \{t_0, t_1\}\, s \mapsto s & \\
\mathbf{S}_i d^\diamond \nu\, \{t_0, t_1\}\, s \mapsto t_i d^\diamond(\nu\, \{t_0, t_1\}\, s) & \nu \text{ ein Numeral} \\
\mathbf{nil}_\tau\, \{t\}\, s \mapsto s & \\
\mathbf{cons}_\tau d^\diamond a\ell\, \{t\}\, s \mapsto td^\diamond a(\ell\, \{t\}\, s) & \ell \text{ eine Liste}
\end{array}
$$

$$
\frac{t \mapsto t'}{t \to t'} \qquad \frac{t \to t'}{ts \to t's} \qquad \frac{s \to s'}{ts \to ts'}
$$