CS-316, Logic and Semantics **Coursework Assignment 2**
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1. Consider a traffic light. We will formalise it using the variables r, y, and g for the events that the red, yellow, and green light, respectively, is on. Provide LTL-formulae $\varphi \in \mathsf{LTL}[r, y, g]$ expressing the following properties.

   - "It is never the case that all three lights are on at the same time."
   - "Every light will be lit at some moment in the future."
   - "Whenever the green light is lit, it will stay on till a yellow light shows."

2. For each of the following $\varphi \in \mathsf{LTL}[a, b, c]$ provide some $\psi \in \mathsf{FOL}[<, A, B, C]$ such that for all word Structures $\mathcal{A}$ we have $\mathcal{A}, t \models \varphi$ if and only if $\mathcal{A}, t \models \psi$.

   - $\mathtt{GF}a$
   - $a\mathtt{U}(b\mathtt{U}c)$
   - $\mathtt{XXX}a$

3. Systematically evaluate the LTL formula $\varphi \equiv (\neg p)\mathtt{U}(\mathtt{XG}q)$ and all its relevant subformulae over the trace structure $\mathcal{A} = (\{1, 2, 3, 4\}, P^{\mathcal{A}}, Q^{\mathcal{A}})$ where $P^{\mathcal{A}} = \{2\}$ and $Q^{\mathcal{A}} = \{3, 4\}$. Does $\mathcal{A}, 1 \models \varphi$?

4. In the lecture we have seen that every LTL φ property can be described by a deterministic finite automaton $\mathcal{M}_{\varphi}$ reading a word backwards. Construct the corresponding automaton over the alphabet $\{0, 1\}^2$ for the $\mathsf{LTL}[p, q]$-formula $(\mathtt{F}p)\mathtt{U}q$. Explain your construction.

5. Consider the two first order structures $\mathcal{A} = (\{1, 2, 3\}, P^{\mathcal{A}}, R^{\mathcal{A}})$ and $\mathcal{B} = (\{1, 2, 3, 4\}, P^{\mathcal{B}}, R^{\mathcal{B}})$ with unary relations $P^{\mathcal{A}} = \{1, 2\}$ and $P^{\mathcal{B}} = \{1, 2\}$, and binary relations $R^{\mathcal{A}} = \{(1, 2)\}$ and $R^{\mathcal{B}} = \{(1, 3)\}$.

   - Give a first-order formula $\varphi \in \mathsf{FOL}[P, R]$ with at most two quantifiers such that $\mathcal{A} \models \varphi$ and $\mathcal{B} \not\models \varphi$.
   - Give an argument that *no* $\varphi \in \mathsf{FOL}[P, R]$ with at most one quantifier can have the property $\mathcal{A} \models \varphi$ and $\mathcal{B} \not\models \varphi$.