We can express the semantics of LTL as a first-order formula speaking about a particular point in time. So we translate a LTL formulae φ into first-order formulae $\tilde{\varphi}(t)$ with one distinguished variable t.

LTL and First-Order Logic

$\varphi \in \mathsf{LTL}[p_1, \ldots, p_n]$	$\tilde{\varphi}(t) \in \mathsf{FOL}[<, P_1, \ldots, P_n]$
p_i	$P_i(t)$
$\mathtt{X}\varphi$	$\exists t'(\chi_{\text{next}}(t, t') \wedge \tilde{\varphi}(t'))$
$\mathtt{F}\varphi$	$\exists t'(t \leq t' \wedge \tilde{\varphi}(t'))$
$\mathtt{G}\varphi$	$\forall t'(t \leq t' \wedge \tilde{\varphi}(t'))$
$\varphi\mathtt{U}\psi$	$\exists t'(\ t \leq t' \wedge \tilde{\psi}(t') \wedge$
	$\forall t''(((t \leq t'') \wedge (t'' < t')) \rightarrow \tilde{\varphi}(t))\ )$

With this translation we get that

$$\mathcal{A}, t \models \varphi \text{ iff } \mathcal{A}, t \models \tilde{\varphi}$$

Note, however, that we couldn't have chosen the trivial translation $\tilde{\varphi} \equiv \varphi$, was we wanted a translation from LTL to first-order logic. Note the different meanings of the $\models$ symbols above!