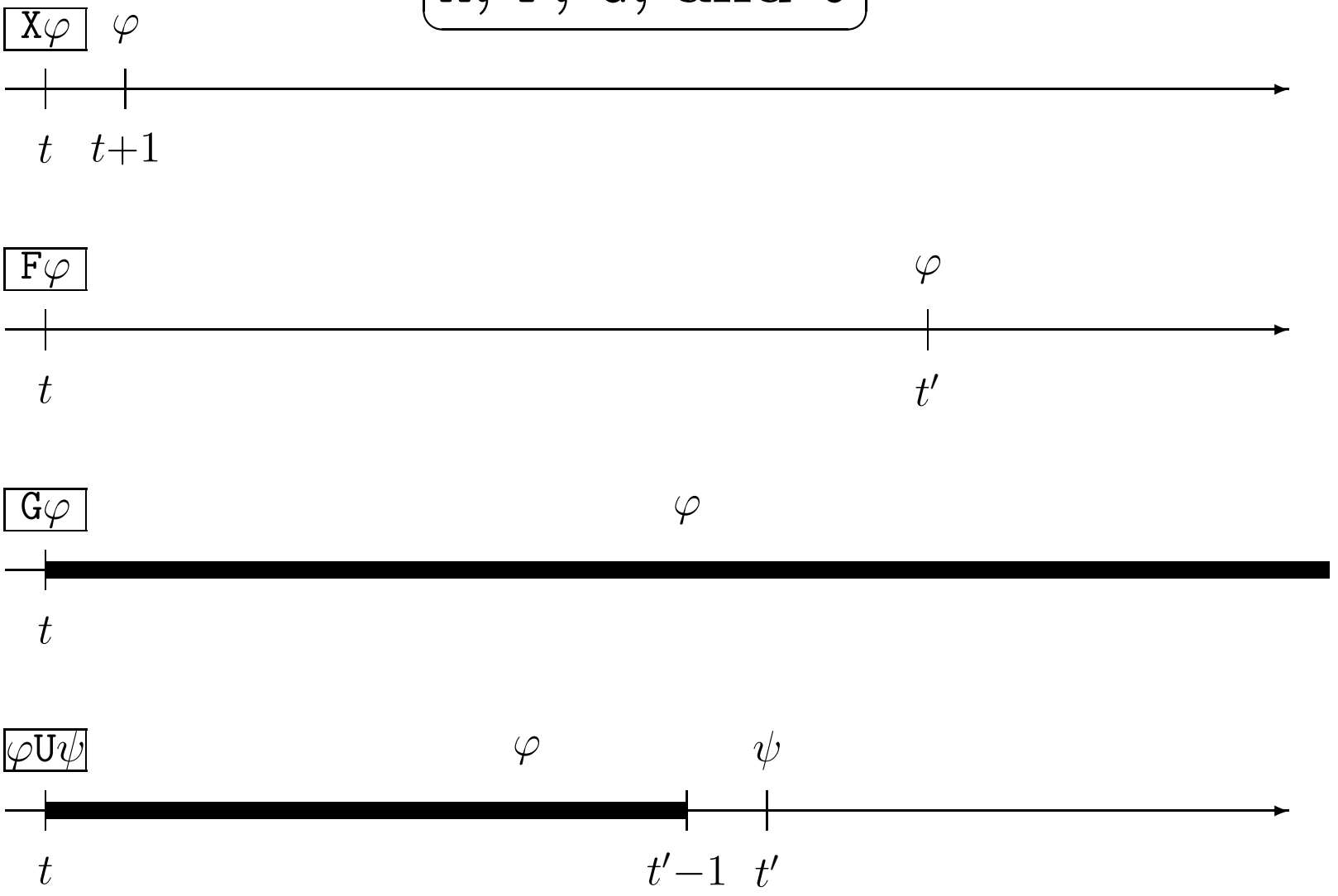
X, F, G, and U



LTL Model Checking

t		t		$t{+}1$
$\mathcal{A}, t \models \mathtt{X}\varphi$	iff			$\mathcal{A}, t{+}1 \models \varphi$
$\mathcal{A}, t \models \mathtt{F}\varphi$	iff	$\mathcal{A}, t \models \varphi$	or	$\mathcal{A}, t{+}1 \models \mathtt{F}\varphi$
$\mathcal{A}, t \models \mathtt{G}\varphi$	iff	$\mathcal{A}, t \models \varphi$	and	$\mathcal{A}, t{+}1 \models \mathtt{G}\varphi$
				in case $t + 1 \leq \ell$
$\mathcal{A}, t \models \varphi \mathtt{U} \psi$	iff	$\mathcal{A}, t \models \psi$	or	
		$\mathcal{A}, t \models \varphi$	and	$\mathcal{A}, t{+}1 \models \varphi \mathtt{U} \psi$

Example for LTL Model Checking

Formula $\mathtt{G}(y \rightarrow y\mathtt{U}r)$. *That this* $\mathtt{G}(\neg y \vee y\mathtt{U}r)$.

$\mathcal{A}$										
	r	1	1	1	0	0	0	0	0	1
	y	0	0	1	0	0	0	1	1	0
	g	0	0	0	1	1	1	0	0	0
$\mathtt{G}(y \rightarrow y\mathtt{U}r)$										
$y \rightarrow y\mathtt{U}r$										
$y\mathtt{U}r$										
$\neg y$										

LTL and Automata

For φ construct DFA $\mathcal{M}_\varphi$ with $L(\mathcal{M}_\varphi) = \{w | w^{-1} \models \varphi\}$.

States: sets of sub-formulae of φ.
indicating with formulae hold at a given position

Transitions: Given

- previous state $\{\psi \mid \mathcal{A}, t{+}1 \models \psi\}$
- t'th letter of w, i.e., local properties of $\mathcal{A}$ at time t

determine $\{\psi \mid \mathcal{A}, t \models \psi\}$ by the rules seen.

First-Order Structures

Definition. A first-order structure $\mathcal{A} = (A, R_1, R_2, \ldots)$ is given by

- A set A, called the "universe" of the structure
- Relations $R_1, R_2, \ldots$ on A.
  I.e., each $R_i \subseteq A^{n_i}$ for some n_i, called the arity.

Syntax of First-Order Logic

The set $\mathsf{FOL}[R_1, \ldots, R_n]$ of first-order formulae over the relation symbols $R_1, \ldots, R_n$ is freely generated as follows.

- $(x = y)$ for variables x, y
- $R_j x_{i_1} \ldots x_{i_{n_j}}$ if R_j is of arity n_j
- $\top$, $\bot$, $\neg\phi$, $(\phi \wedge \psi)$, $(\phi \vee \psi)$ for formulae ϕ, ψ
- $\forall x\varphi$ and $\exists x\varphi$ for ϕ a formula and x a variable