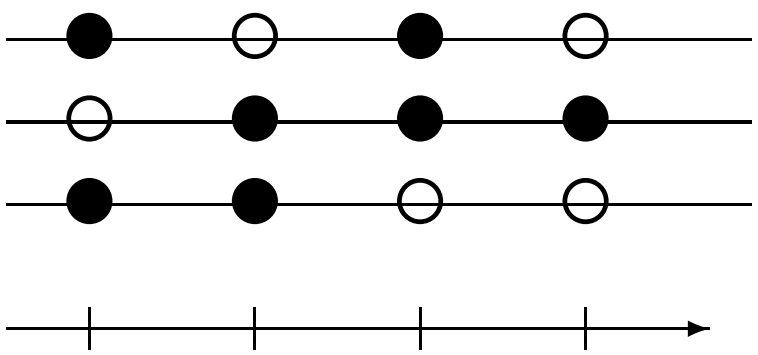
Finite Trace Structures



$R = \{1, 3\}$

$Q = \{2, 3, 4\}$

$P = \{1, 2\}$

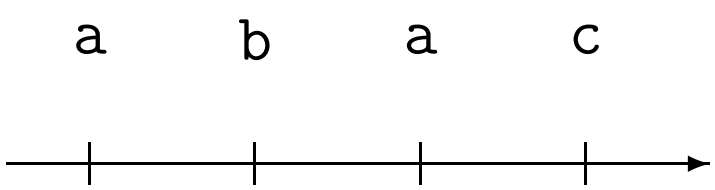
1 2 3 4

Universe $\{1, 2, 3, 4\}$

$<= \{(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)\}$

Word Structures

$P_{\mathtt{a}} = \{1, 3\}$, $P_{\mathtt{b}} = \{2\}$, $P_{\mathtt{c}} = \{4\}$



Universe $\{1, 2, 3, 4\}$

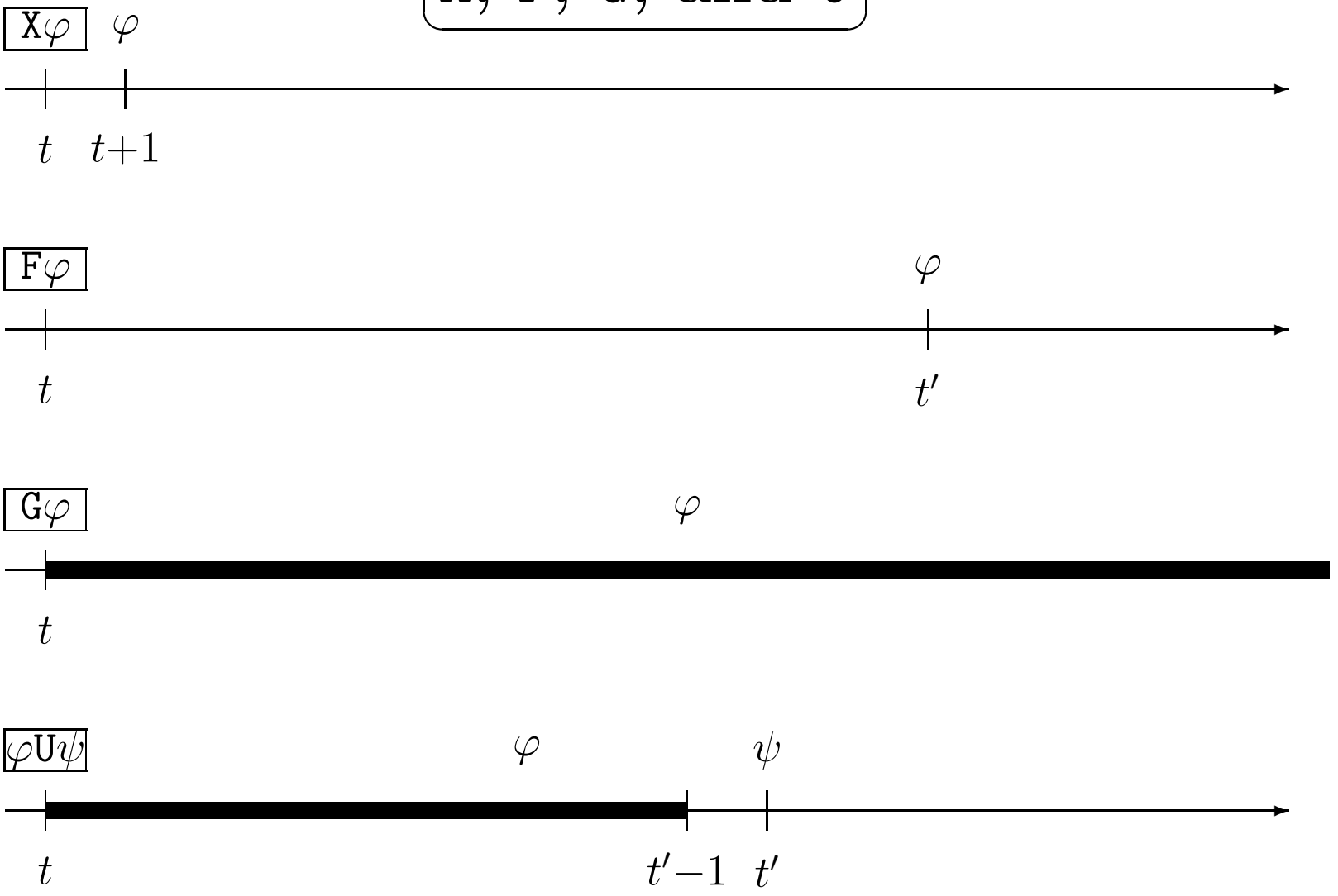
$<= \{(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)\}$

Syntax of Linear Temporal Logic

The set $\mathsf{LTL}[p_1, \ldots, p_n]$ of *LTL-formulae* is freely generated

- $\top$, $\bot$, and all $p_i \in \{p_1, \ldots, p_n\}$
- If $\varphi, \psi \in \mathsf{LTL}[p_1, \ldots, p_n]$, then also $\neg\varphi$, $(\varphi \wedge \psi)$, $(\varphi \vee \psi)$.
- . . . and also
  - $\mathtt{X}\varphi$ "next"
  - $\mathtt{F}\varphi$ "finally"
  - $\mathtt{G}\varphi$ "globally"
  - $(\varphi \mathtt{U} \psi)$ "until"

X, F, G, and U



Semantics of LTL

Let $\mathcal{A} = (\{1, \ldots, \ell\}, <, P_1, P_2, \ldots, P_n)$ and $t \in \{1, \ldots, \ell\}$.
Define $\mathcal{A}, t \models \varphi$ for $\varphi \in \mathsf{LTL}[p_1, \ldots, p_2]$ inductively.

- $\mathcal{A}, t \models \top$; $\mathcal{A}, t \not\models \bot$; $\mathcal{A}, t \models p_i$ iff $t \in P_i$
- $\mathcal{A}, t \models \varphi \wedge \psi$ iff ...
- $\mathcal{A}, t \models \mathtt{X}\varphi$ iff $t < \ell$ and $\mathcal{A}, t{+}1 \models \varphi$
- $\mathcal{A}, t \models \mathtt{F}\varphi$ iff $\mathcal{A}, t' \models \varphi$ for some $t' \geq t$
- $\mathcal{A}, t \models \mathtt{G}\varphi$ iff $\mathcal{A}, t' \models \varphi$ for all $t' \geq t$
- $\mathcal{A}, t \models \varphi \mathtt{U} \psi$ iff there is some $t' \geq t$ s.t.
  $\mathcal{A}, t' \models \psi$ and $\mathcal{A}, t'' \models \varphi$ for all $t \leq t'' < t'$

Model Relation for Propositional Logic

Writing $\underline{a} \models \varphi$ for $\varphi[\underline{a}] = 1$ we obtain the following.

- $\underline{a} \models \top$ always holds and $\underline{a} \models \bot$ never holds
- $\underline{a} \models \neg\varphi$ holds iff $\underline{a} \models \varphi$ does *not* hold
- $\underline{a} \models \varphi \wedge \psi$ holds if $\underline{a} \models \varphi$ *and* $\underline{a} \models \psi$ both hold.
  $\underline{a} \models \varphi \vee \psi$ holds if $\underline{a} \models \varphi$ holds *or* $\underline{a} \models \psi$ holds.

Example

Consider a traffic light. In our formalisation, we will use the variables r, y, g for the events the red/yellow/green light is on.

Formalise the following events.

- "There is always at least one light on"
- "It is always the case, that you will get a green light sometimes"
- "Whenever there's a yellow light, it will stay till a red light shows up"

Example for LTL Model Checking

Formula $\mathtt{G}(y \rightarrow y\mathtt{U}r)$. *That this* $\mathtt{G}(\neg y \vee y\mathtt{U}r)$.

$\mathcal{A}$										
	r	1	1	1	0	0	0	0	0	1
	y	0	0	1	0	0	0	1	1	0
	g	0	0	0	1	1	1	0	0	0
$\mathtt{G}(y \rightarrow y\mathtt{U}r)$										
$y \rightarrow y\mathtt{U}r$										
$y\mathtt{U}r$										
$\neg y$										

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LTL and Automata

For φ construct DFA $\mathcal{M}_\varphi$ with $L(\mathcal{M}_\varphi) = \{w | w^{-1} \models \varphi\}$.

States: sets of sub-formulae of φ.
indicating with formulae hold at a given position

Transitions: Given

- previous state $\{\psi \mid \mathcal{A}, t{+}1 \models \psi\}$
- t'th letter of w, i.e., local properties of $\mathcal{A}$ at time t

determine $\{\psi \mid \mathcal{A}, t \models \psi\}$ by the rules seen.