Run of an Automaton

Let $\mathfrak{A} = (Q, I, \Delta, F)$ be an NFA, $Q = \{q_0, \ldots, q_n\}$.

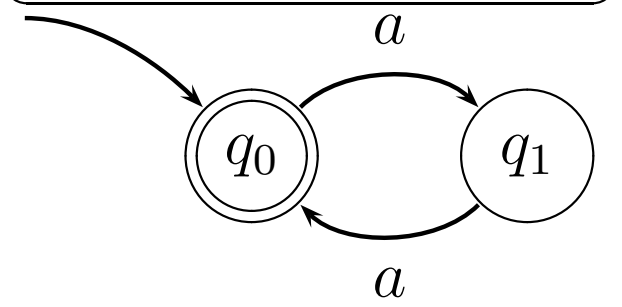
"$\mathfrak{A}$ has an accepting run": $\exists X_0 \ldots \exists X_n(\varphi_i \wedge \varphi_s \wedge \varphi_f)$

$$\varphi_i \equiv \forall x(\text{“}x \text{ first”} \rightarrow \bigwedge_j (X_j(x) \rightarrow \bigvee_{q_i \in I, (q_i,a,q_j) \in \Delta} P_a(x)))$$

$$\varphi_s \equiv \forall x \forall y(\chi_{\mathsf{next}}(x,y) \rightarrow \bigwedge_j (X_j(y) \rightarrow \bigvee_{(q_i,a,q_j \in \Delta)} (X_i(x) \wedge P_a(y))))$$

$$\varphi_f \equiv \forall x(\text{“}x \text{ last”} \rightarrow \bigvee_{q_j \in F} X_j(x))$$

Example: $(aa)^*$



$$\exists X_0 \exists X_1 [\ \forall x((\forall y.x \leq y) \rightarrow (X_1(x) \wedge P_a(x))) \wedge$$
$$\forall x \forall y(((x < y) \wedge \neg \exists z(x < z \wedge z < y)) \rightarrow$$
$$((X_0(y) \rightarrow (X_1(x) \wedge P_a(y))) \wedge$$
$$(X_1(y) \rightarrow (X_0(x) \wedge P_a(y))))) \wedge$$
$$\forall x((\forall y.y \leq x) \rightarrow X_0(y))]$$