CS Questions Around Formalisms

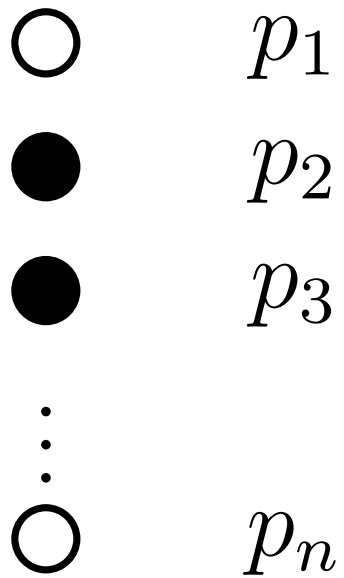
Does my program satisfy the spec?
Is there any program satisfying it?

} How difficult to check?

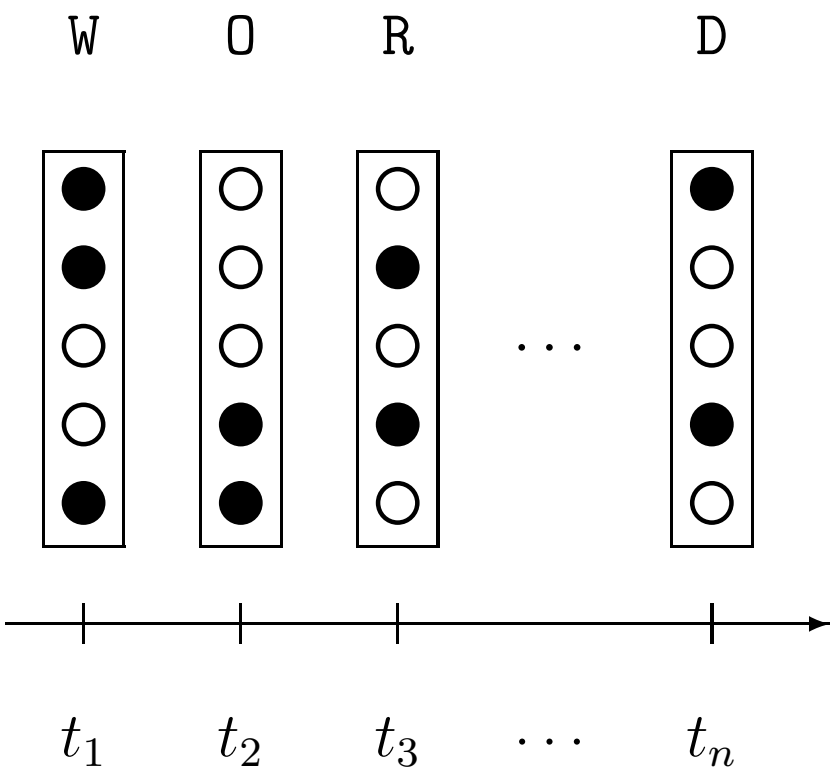
Can I express the property at all?

. . . and how complicated?

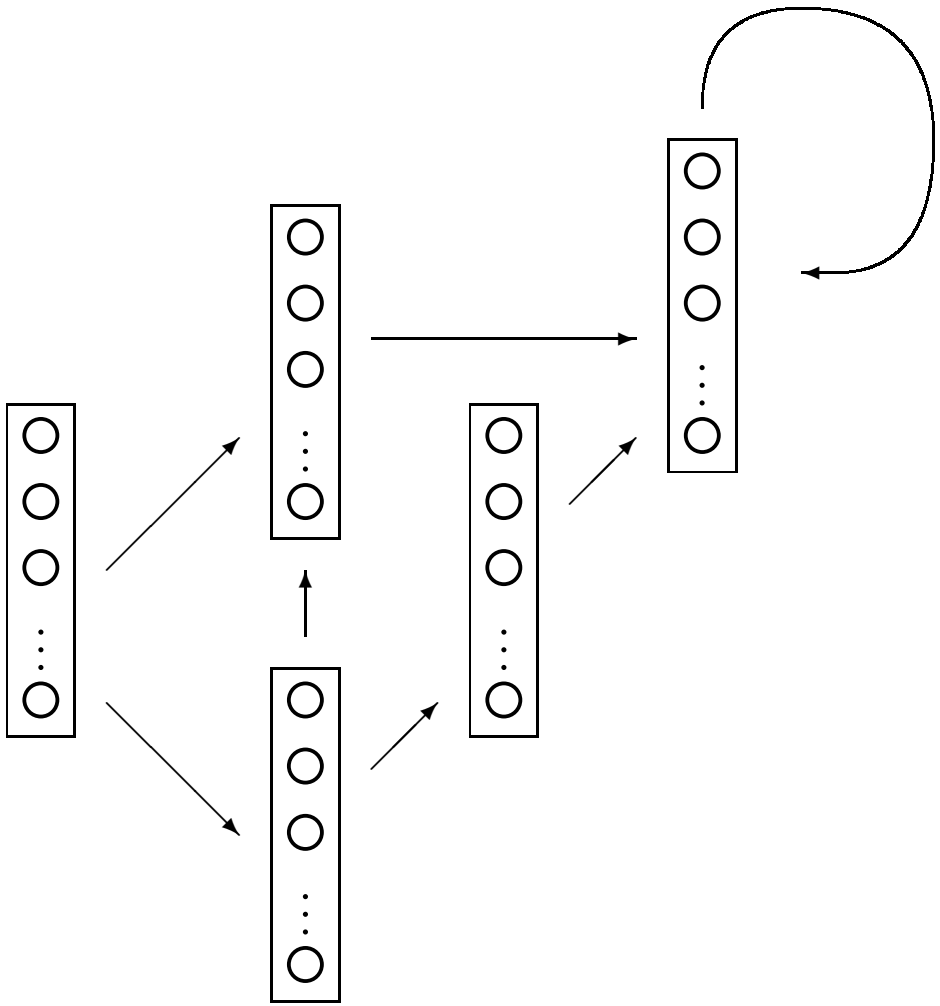
Propositional Logic



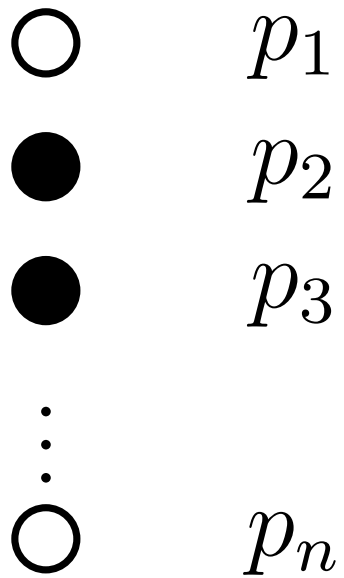
First-Order Logic



Modal Logic



Propositional Logic



Syntax of Propositional Logic

The set $\mathsf{PL}[p_1, \ldots, p_n]$ of *propositional formulae over* $p_1, \ldots, p_n$ is freely generated as follows.

- $\top$, $\bot$, and all $p_i \in \{p_1, \ldots, p_n\}$ are propositional formulae *(so called "atomic formulae").*
- If φ is a propositional formula, then so is $\neg\varphi$.
- If φ and ψ are propositional formulae, then so are $(\varphi \wedge \psi)$ and $(\varphi \vee \psi)$.

Evaluation of a Boolean Formula

For $\varphi \in \mathsf{PL}[p_1, \ldots, p_n]$ and $\underline{\mathrm{a}} = (a_1, \ldots, a_n) \in \{0,1\}^n$ we define $\varphi[\underline{\mathrm{a}}] \in \{0,1\}$ as follows.

- $\top[\underline{\mathrm{a}}] = 1$, $\bot[\underline{\mathrm{a}}] = 0$, $p_i[\underline{\mathrm{a}}] = a_i$
- $(\neg\varphi)[\underline{\mathrm{a}}] = \curlywedge(\varphi[\underline{\mathrm{a}}])$
- $(\varphi \wedge \psi)[\underline{\mathrm{a}}] = (\varphi[\underline{\mathrm{a}}]) \mathbb{\wedge} (\psi[\underline{\mathrm{a}}])$,
  $(\varphi \vee \psi)[\underline{\mathrm{a}}] = (\varphi[\underline{\mathrm{a}}]) \mathbb{\vee} (\psi[\underline{\mathrm{a}}])$

The functions $\curlywedge, \mathbb{\wedge}, \mathbb{\vee}$ are given by

x	y	$\curlywedge y$	$x \mathbb{\wedge} y$	$x \mathbb{\vee} y$
0	0	1	0	0
0	1	0	0	1
1	0		0	1
1	1		1	1